Entrepreneurship, On-the-job Search and Informal Jobs *

GAETANO LISI

Department of Economics, University of Cassino, Italy

ABSTRACT This paper develops a search and matching model of equilibrium unemployment, with on-the-job search, extended to both the informal sector and entrepreneurship. Three are the key features of this model: the entrepreneurial ability affects job productivity, all unemployed start their job search in the official sector, and workers employed in the informal sector try to move into the official one. Two key results emerge from the analysis. First: firms become heterogeneous in productivity, thus providing a new solution to the problem of finding an interior equilibrium where vacant jobs are allocated to both the regular and the hidden sector (the so-called 'shadow puzzle'). Second: if the informal sector is sufficiently large, an increase in labour market tightness increases the unemployment rate and then the 'vacancies-unemployment' relationship (the so-called 'Beveridge Curve') switches from negative to positive.

JEL CLASSIFICATION E26, J23, J24, J63, J64, L26

KEYWORDS entrepreneurship, on-the-job search, unemployment, matching models, shadow/hidden/underground/informal economy

* A previous version of this paper was presented at the Third Italian Doctoral Workshop in *Economics and Policy Analysis* (University of Torino and Collegio Carlo Alberto, Moncalieri, Torino, July 8-9, 2010), and accepted at the *XXV National Conference of Labour Economics* (Chieti-Pescara, 9-10 September, 2010).

Correspondence Address: Gaetano Lisi, Department of Economic Sciences, University of Cassino, via S. Angelo, I-03043 Cassino (FR), Italy. Email: gaetano.lisi@unicas.it.

1. Introduction

This paper suggests an extension of the matching model of equilibrium unemployment with informal sector which takes into account the heterogeneity in entrepreneurial ability (and hence productivity of jobs) and the possibility of on-the-job search from informal sector to the official one. The value added of the paper is that the three extensions of the basic matching model (informal sector, on-the-job search and heterogeneity in entrepreneurial ability) are put together.

The approach allows coming up with an explanation of the persistence of the informal or underground economy in equilibrium (the so-called "shadow puzzle"). Indeed, the underground economy is persistent also in high-income OECD countries in spite of improvements in technologies detecting tax evasion. The underground share of GDP ranges from a low 10 per cent of GDP in the Nordic countries, UK and Switzerland, to peaks of 20 to 30 per cent in Southern Europe and Ireland and 40 per cent in transitional economies of Eastern Europe and Asia.

The present paper addresses this important issue and offers a simple theoretical explanation of the "shadow puzzle": heterogeneous entrepreneurs sort themselves into official or unofficial sector according to their entrepreneurial ability levels. In short, the less able entrepreneurs enjoy tax evasion and create vacancies/employment in the informal sector where the entrance barriers (the start-up costs) are lower. Indeed, the ease of entry is often used in literature as one of the key criteria for defining the informal sector.

Since the informal jobs report much higher separation rates (informal activities are in fact detected and repressed by the government), all workers prefer to work in the official sector in order to preserve their job. Nevertheless, because of high unemployment and skill-mismatch problems, it is often not convenient to reject informal job offers and wait for a job offer from the official sector. As a result, workers employed in informal jobs try to move into official ones.

In addition, the model generates a positively-sloped Beveridge curve within a reasonable value of labour market tightness and interval of parameters (monitoring and job destruction rates), thus enriching our understanding of the vacancies-unemployment relationship under different sets of parameters. In developing countries, informal firms account for about half of all economic activity. According to the model, the two sectors are complementary (i.e. if the informal vacancies increase the official vacancies decrease and vice versa), and the unemployment rate is adversely affected by vacancies, both informal and official. Hence, if the share of entrepreneurs

who enter the underground sector is sufficiently large (say, equal to or higher than the official one), then it is the effect of informal sector which prevail, namely, an increase in informal vacancies decreases both official vacancies and unemployment. Consequently, the relationship between labour market tightness and unemployment (well-known in literature as the "Beveridge curve") switches from negative to positive, since labour market tightness positively depends on official vacancies. This result may explain why the informal sector has been tolerated in some countries, in spite of its adverse effects on tax revenues.

The rest of this paper is organized as follows: section 2 underlines the novelties of the paper; section 3 presents a two-sector matching model with on-the-job search, informal jobs and heterogeneous entrepreneurs; while section 4 provides a comparative statics exercise which summarizes the behaviour of the model following changes in the productivity of the match; finally, section 5 concludes.

2. Matching models and the underground economy

Usually in matching-type models, the supply side of the labour market is well-characterized, whereas the demand side is described only by the free-entry (or zero-profit) condition used to determine the vacancies posted by firms. For instance, Albrecht et al. (2009) and Boeri and Garibaldi (2006) model the heterogeneous ability of workers which affects job productivity. However, analogous reasoning should also be applied to the (entrepreneurial) ability of entrepreneurs. Nevertheless, the additional extension of matching models to entrepreneurship is in fact new, as pointed out by Lisi and Pugno (2010). Indeed, also Fonseca et al. (2001) and Pissarides (2002) have pursued this kind of analysis, but they have ignored the fact that entrepreneurial ability affects job productivity. In particular, in their models the entrepreneurship only affects the entry costs. This is a shortcoming for the benchmark macroeconomic model of the labour market since entrepreneurship is crucial for spurring structural change, new firms, innovations and employment growth (Belke et al., 2003). Entrepreneurship is in fact like a production factor, the so-called "entrepreneurial capital" (Audretsch and Keilbach, 2004; Audretsch, 2007), and then it realistically affects job productivity.

Furthermore, in many analyses, the *on-the-job* search assumption is often neglected. Although the equilibrium unemployment theory obtained in matching models with *on-the-job* search is not significantly different from that obtained without it (see Pissarides, 2000), the *on-the-job* search assumption is quite realistic since the match is not the end of the job search. The goal of search activity is in fact to find "*the best*

partner not a simple partner". This is even more relevant when the underground economy is taken into account, since it is seen as a backward and less productive economy, in which the wages are (often) lower. Nevertheless, because of high unemployment and skill-mismatch problems, it is often not convenient to reject informal job offers and wait for a job offer from the official sector. However, once hired, workers try to move from the informal sector to the official one.

This theoretical paper belongs to a growing literature which has just put together the underground economy theory with the benchmark macroeconomic model of the labour market, i.e. the search and matching model (Bouev, 2002, 2005; Boeri and Garibaldi, 2002, 2006; Kolm and Larsen, 2003, 2010; Fugazza and Jacques, 2004; Albrecht et. al., 2009; Bosch and Esteban-Pretel, 2009). Among this literature, this model is perhaps most related to that of Boeri and Garibaldi (2002) and Bosch and Esteban-Pretel (2009), who make use of a matching model extended to the underground sector with the possibility of direct transitions from informal into formal employment. However, neither of them take into account the entrepreneurial ability on the demand side. Furthermore, in this model, matching first takes place for official jobs and then for informal jobs, which means each worker prefers to work in the official sector (for instance, because of higher wages, lower separation rates or in order to be entitled to a pension). Indeed, the presence of this "search path" followed by all unemployed workers is a novelty in the matching literature with respect to the standard assumptions of *directed* or *random* search.[1]

More precisely, this paper adopts a search and matching model of equilibrium unemployment *à la* Pissarides (2000), with *on-the-job* search, extended to the underground sector and to entrepreneurial ability. These three features/extensions together have interesting analytical consequences. The key ones are two. First: firms become heterogeneous in productivity, thus providing a new solution to the problem of determining a mixed allocation of vacant jobs between the regular and the irregular sector (the so-called *shadow puzzle*, i.e. the persistence of the underground economy in equilibrium). Second: if the informal sector is very large (as in several developing countries),[2] an increase in labour market tightness increases the unemployment rate and then the 'vacancies-unemployment' relationship (the so-called *Beveridge Curve*)

[1] In short, the unemployed workers select the sector in which searching for a job (*directed* search) or they take the first available job (*random* search). Dulleck et al. (2006) make use of a simple and modified version of the search model in which matching takes place first for high-skilled jobs and then for low-skilled jobs. However, in their model there is not the underground economy and the unemployment analysis.

[2] Indeed, in developing countries, informal firms account for about half of all economic activity (La Porta and Shleifer, 2008).

switches from negative to positive. In general, the ambiguity of this result reflects both the different trend of the two economic sectors (Busato and Chiarini, 2004) and the intricate relationship found in the literature between unemployment and underground employment. For instance, according to Bouev (2002, 2005), scaling down the unofficial sector can lead to a decrease in the level of unemployment; whereas according to Boeri and Garibaldi (2002, 2006), attempts to reduce, in the first place, shadow employment will result in higher open unemployment.[3]

3. The model

We propose a search and matching model with on-the-job search and informal jobs. The two-sector economy is populated by a [0,1] continuum of entrepreneurs (one-job firms) and a [0,1] continuum of workers.[4] Entrepreneurs can either operate regularly or against the tax regulations, i.e. evading taxes. Since the informal activities are detected and repressed by the government, all workers prefer to work in the official sector in order to preserve their job. Indeed, informal jobs report much higher separation rates (Bosch and Esteban-Pretel, 2009). Hence, workers employed in informal jobs try to move into official ones.[5]

The matching frictions on the official side of the labour market are captured by a popular constant returns to scale (*CRS*) matching function (Pissarides, 2000; Petrongolo and Pissarides, 2001):

$$m_r = m\{v_r, u + n_s\} \Rightarrow \theta \equiv \frac{v_r}{u + n_s}$$

where θ is the labour market tightness, v_r is the number of vacancies supplied by official firms, u is the unemployment rate and n_s is the shadow employment rate, i.e. the measure of employed job-seekers. The subscript $i \in \{r, s\}$ denotes the sector, where r = *regular* and s = *shadow*.

A crucial and novel assumption related to the preference of workers is introduced: only the unemployed workers who fail to find a job in the official sector search in the informal one. Hence, this implies that matching initially takes place for official jobs and then for informal jobs. Therefore, the share of job-seekers in the

[3] Indeed, the sign of this relationship depends on various assumptions regarding the position of shadow employment in the Labour Force Statistics (Boeri and Garibaldi, 2002, 2006) since very heterogeneous workers compose the labour force of hidden economy (Tanzi, 1999).

[4] The use of a different set for entrepreneurs and workers would not change the qualitative results of the analysis.

[5] Time is continuous, and individuals are risk neutral and infinitely lived. We neglect possibilities of moonlighting, so workers can perform only one activity at a time.

informal sector is equal to $\tilde{u} \equiv u \cdot [1 - g(\theta)dt]$, since the unemployed worker fails to find a job in the official sector with probability $1 - g(\theta)dt$. The instantaneous probability of finding an official job, $g(\theta)$, has the following standard properties: $g'(\theta) > 0$, $g''(\theta) < 0$, and $\lim_{\theta \to 0(\infty)} g(\theta) = 0\,(\infty)$. Furthermore, we assume frictionless matching for informal jobs, i.e. that there is a spot-market for informal jobs.[6] The number of informal matches is thus given by:

$$m_s = min\{\tilde{u}, v_s\}$$

where v_r is the number of informal vacancies.

As stated above, job search takes place in two sequential steps: at first, all unemployed workers search in the official sector, and afterwards (in the case of failure) they search in the informal one. Hence, the value of searching for a job *(U)* is given by: [7]

$$rU = l - k + g(\theta) \cdot [W_r - U] + [1 - g(\theta)] \cdot \gamma \cdot [W_s - U]$$

where r is the discount rate; l is the value of leisure; k is the search cost of unemployed workers; $\gamma \equiv min\{\tilde{u}, v_s\} / \tilde{u}$ is the probability of finding an informal job, and W_i is the value for being employed:

$$rW_r = w_r + \delta \cdot [U - W_r]; \qquad rW_s = w_s + (\delta + \rho) \cdot [U - W_s] + g(\theta) \cdot [W_r - W_s]$$

where w_i is the wage rate; δ is the exogenous job destruction rate; and ρ is the exogenous probability of a firm being discovered and destroyed as unregistered. As in Pissarides (2000), it is implicitly assumed that employed job-seekers and unemployed workers search with the same intensity and that they are equally good at finding official jobs. Hence, official jobs arrive to each job-seeker at the same rate, which is equal to $g(\theta)$.

Finally, given the assumption of matches without frictions in the informal sector, if $v_s \geq \tilde{u}$ all unemployed workers find a job in their "search path", i.e. the outflow from the unemployment pool is exactly equal to the unemployment rate $(u - u \cdot g(\theta) - u \cdot [1 - g(\theta)] \cdot \gamma$ is in fact zero if $\gamma = 1)$. Since the search frictions in the official sector could cause a small (large) number of matches (unemployed), a non-

[6] Zenou (2008) also develops a labour market model in which the formal sector is subject to search frictions, whereas the informal sector is competitive. Indeed, the informal sector is a "freewheeling" sector with respect to the official one (consider the firing cost in the official sector, which discourages the recruiting of personnel or the "red tape", which delays hiring).

[7] The search "timing" – namely, unemployed workers never search in the informal sector before failing to find a job in the official one – implies that the two steps of the "search path" are independent.

trivial result requires that $\widetilde{u} > v_s$.[8] Hence, the probability of filling an informal vacancy is equal to 1 and the bargaining power of workers who search for an informal job is zero. As a result, the informal wage is equal to a given minimum wage b, with $w_r > b \equiv w_s$. $W_r > W_s > U$ is in fact a necessary condition for a consistent equilibrium.[9]

3.1 Entrepreneurship

In this section, we follow Lisi and Pugno (2010). Entrepreneurs are born with a specific and positive entrepreneurial ability x which is drawn from a known distribution, $F : [x_{min}, x_{max}] \rightarrow [0,1]$, and affects the job productivity:

$$rV_r = -c + q(\theta)\cdot[J_r - V_r]; \qquad (r+\delta)\cdot J_r = xp - w_r - \tau$$

$$rV_s = [J_s - V_s], \qquad [r+\delta+g(\theta)]\cdot J_s = xp\phi - b + \rho\cdot[0 - J_s - \varphi\tau]$$

where V_i is the value of a vacancy; J_i is the value of a filled job (from the informal sector stand-point, on-the-job search operates as an increase in the discount rate, since it reduces the average duration of employment); c is the start-up cost (which is zero in the informal sector); x is the entrepreneurial ability; τ is an exogenous production tax; p is the productivity of the match; while $q(\theta)$ refers to the instantaneous probability of filling a vacancy in the official sector, with $q'(\theta) < 0$, $q''(\theta) > 0$, and $\lim_{\theta \to 0(\infty)} q(\theta) = \infty(0)$. The parameter $\phi \in (0,1)$ captures the fact that the informal sector generally utilises less efficient technology; indeed, underground activities are often seen as *labour intensive* activities (Busato and Chiarini, 2004). Furthermore, if discovered as unregistered, the informal firm pays a large penalty $\varphi\tau$, with $\varphi > 1$.

A successful official match has net productivity equal to $(xp - \tau)$. We assume that official wages are given by $\beta\cdot(xp - \tau)$, where β is the bargaining power of workers.[10] To ensure that official production takes place we also assume $(1-\beta)\cdot(xp - \tau) > c$. If this did not hold true, there would be no official jobs, which is a trivial case.

---

[8] This condition could remain unsatisfied when unemployment is very small, i.e. the probability of finding an official job is very large. But in this case the underground economy would be a negligible phenomenon.

[9] Wages in "bad" (informal) jobs do not depend on outside market conditions. This is a standard feature of matching models with *on-the-job* search, as noted by Pissarides (2000) and Boeri and Garibaldi (2002).

[10] This is the solution to a non-cooperative bargain game of the Rubinstein type where it is impossible to search while negotiating (see e.g. Mortensen, 2005). In the standard models (see Pissarides, 2000), the outside option is typically assumed to be unemployed search. The standard specification adds complexity but no further insight.

The cut-off condition which defines a threshold level of entrepreneurial ability, $R \in [x_{min}, x_{max}]$, such that the marginal entrepreneur is indifferent to operating in the informal or formal sector is the following (*entrepreneurs' indifference condition*):

$$V_r(x = R) = V_s(x = R) \tag{1}$$

hence, R can be derived in a straightforward manner (see *Appendix A*):

$$R = \frac{\dfrac{c}{[r + q(\theta)]} + \Omega(\theta) \cdot \tau - \dfrac{b + \rho \cdot \varphi\tau}{\Lambda(\theta)}}{p \cdot \left[\Omega(\theta) - \dfrac{\phi}{\Lambda(\theta)}\right]} \tag{2}$$

with $\dfrac{q(\theta) \cdot (1 - \beta)}{(r + \delta) \cdot [r + q(\theta)]} \equiv \Omega(\theta)$, and $(1 + r) \cdot [r + \delta + \rho + g(\theta)] \equiv \Lambda(\theta)$. The restrictions which ensure the positivity of R (see *Appendix A*) imply that the intercept of $V_r(x)$ is more negative than the intercept of $V_s(x)$, and that the slope of $V_r(x)$ is steeper than the slope of $V_s(x)$. This means that (see figure 1),

========== *Figure 1 about here (now at the end)* ==========

Remark 1. *Official jobs are managed by the more able entrepreneurs.*

Since for $x > R \Rightarrow V_r > V_s$, while for $x < R \Rightarrow V_s > V_r$. This key result is consistent with the standard assumption that informal jobs are low productivity jobs (see e.g. Boeri and Garibaldi, 2002, 2006; Kolm and Larsen, 2010). Furthermore, the key property $\partial R / \partial \theta > 0$ (see *Appendix A*) is very intuitive since the higher the labour market tightness, the more difficult it is to fill a regular vacancy and thus more entrepreneurs enter the informal sector.

Given the remark 1, the *entrepreneurs' indifference condition* implies that the share of entrepreneurs (either posting a vacancy or producing) in the hidden sector is $F(R) = n_s + v_s$, while the share $1 - F(R) = n_r + v_r$ runs a business in the official sector. Using the *unemployment identity* or *summing-up* condition on the supply side, i.e. $u = 1 - n_r - n_s$, the aggregate definition of labour market tightness thus becomes the following (see *Appendix B*):

$$\theta = \frac{\{[1 - F(R)] - n_r\}}{u + n_s} = \frac{\{[1 - F(R)] - n_r\}}{1 - n_r} \Rightarrow \theta \cdot \delta + g(\theta) \cdot F(R) = [1 - F(R)] \cdot \delta \tag{3}$$

with $\partial\theta / \partial R < 0$, since at higher R more entrepreneurs enter the informal sector and less vacancies are opened in the official one. Hence, equations (2) and (3) can be represented in the same diagram with axes $[\theta, R]$, as in fig. 2. As a result,

========== *Figure 2 about here (now at the end)* ==========

Remark 2. *There is a unique couple of* (θ, R) *in this two-sector economy.*

The two key variables of the model, i.e. the equilibrium value of labour market tightness and the threshold value of entrepreneurial ability, can thus be obtained.

3.2 The "Beveridge curve"

The steady-state equilibrium conditions determining the employment rates n_r and n_s are given by:

$$\underbrace{g(\theta)\cdot(1-n_r)}_{\text{inflow}}=\underbrace{\delta\cdot n_r}_{\text{outflow}} \Rightarrow n_r=\frac{g(\theta)}{\delta+g(\theta)} \tag{4}$$

since $(1-n_r)=u+n_s$;

$$\underbrace{v_s}_{\text{inflow}}=\underbrace{[\delta+\rho+g(\theta)]\cdot n_s}_{\text{outflow}} \Rightarrow n_s=\frac{v_s}{[\delta+\rho+g(\theta)]}$$

since $\gamma\cdot\tilde{u}=v_s$. The total share of entrepreneurs in the informal sector is $F(R)=n_s+v_s$. Hence, we get:

$$v_s=\frac{F(R)\cdot[\delta+\rho+g(\theta)]}{[\delta+\rho+g(\theta)+1]} \tag{5}$$

$$n_s=\frac{F(R)}{[\delta+\rho+g(\theta)+1]} \tag{6}$$

Finally, using the *summing-up* condition or *unemployment identity*, we obtain the unemployment equation, i.e. the *Beveridge curve*, of this economy:

$$u=\frac{\delta}{[\delta+g(\theta)]}-\frac{F(R)}{[\delta+\rho+g(\theta)+1]} \tag{7}$$

note that $\lim_{\rho\to\infty} n_s=0$. But, on the other hand, the steady-state unemployment rate would be higher, since $\lim_{\rho\to\infty} u=\delta/[\delta+g(\theta)]$. This result explains why governments may be reluctant to repress the informal sector (Boeri and Garibaldi, 2006).

The policy implications of the model can be drawn from the effects of changes in the policy parameters τ, c and ρ on R (since the higher R the larger the share of informal entrepreneurs):

$$\frac{\partial R}{\partial\tau}>0\left(\frac{\partial\theta}{\partial\tau}<0\right); \qquad \frac{\partial R}{\partial c_r}>0\left(\frac{\partial\theta}{\partial c_r}<0\right)$$

in other words, lower taxation and lower start-up cost in the official sector reduce the informal sector and increases labour market tightness. These results are intuitive and

standard in literature (see e.g. Bouev, 2005; Boeri and Garibaldi, 2002, 2006). Conversely, closer monitoring reduces the informal sector but has an *a priori* ambiguous effect on labour market tightness, since $\partial u / \partial \rho > 0$.

4. Comparative Statics: changes in productivity of the match

This section presents a comparative statics exercise which summarizes the qualitative predictions of the model and provides insights for the behaviour of the model in steady state following changes in the productivity of the match.

The equilibrium of the model is characterized by two key variables, θ and R, which satisfy the following two equations:

$$R = \frac{\dfrac{c}{[r+q(\theta)]} + \Omega(\theta)\cdot\tau - \dfrac{b+\rho\cdot\varphi\tau}{\Lambda(\theta)}}{p\cdot\left[\Omega(\theta) - \dfrac{\phi}{\Lambda(\theta)}\right]} \tag{I}$$

$$\theta\cdot\delta + g(\theta)\cdot F(R) = [1-F(R)]\cdot\delta \tag{II}$$

The remaining important variables of the model, (n_r, n_s, v_s, u), formal and informal employment, informal vacancies and unemployment are given by the following four equations:

$$n_r = \frac{g(\theta)}{\delta + g(\theta)} \tag{III}$$

$$n_s = \frac{F(R)}{\delta + \rho + g(\theta) + 1} \tag{IV}$$

$$v_s = \frac{F(R)\cdot[\delta + \rho + g(\theta)]}{\delta + \rho + g(\theta) + 1} \tag{V}$$

$$u = \frac{\delta}{[\delta + g(\theta)]} - \frac{F(R)}{[\delta + \rho + g(\theta) + 1]} \tag{VI}$$

Figure 3 illustrates graphically the implication of an increase in productivity, *p*.

========== *Figure 3 about here (now at the end)* ==========

Proposition 1. *An increase in productivity, p, reduces the threshold value of entrepreneurial ability and increases the labour market tightness. Hence, more entrepreneurs enter the official sector.*

The productivity change only affects equation (I). Differentiating it with respect to p yields:

$\frac{\partial R}{\partial p} < 0$, with $\left[\Omega(\theta) - \frac{\phi}{\Lambda(\theta)}\right] > 0$ which is nothing but a condition for the positivity of R (see *Appendix A*). In short, in times of economic growth, regular firms benefit more from the increase in productivity, because of a more efficient technology.

Proposition 2. *An increase in productivity, p, increases the probability of finding an official job. Hence, it increases the share of official employment but reduces the share of informal employment.*

These effects can be obtained in a straightforward manner from equations (III) and (IV). Therefore, the share of formal employment follows a pro-cyclical pattern, whereas the share of informal employment follows a counter-cyclical pattern.

Proposition 3. *In turn, the previous effect increases the level of informal vacancies.*

This follows from equation (V), since $\partial v_s / \partial g(\theta) > 0$. Indeed, for the informal firms the probability of finding a job in the official sector is a further destruction rate. Hence, the higher $g(\theta)$, the larger the direct transitions of workers from the informal sector to the official one, i.e. the higher the destruction of informal employment n_s. Hence, informal jobs become vacant once again.

Proposition 4. *Eventually, the net effect of labour market tightness on unemployment is a priori ambiguous.*

An increase in labour market tightness reduces informal employment but at the same time it spurs informal vacancies (see Propositions 2 and 3). Hence, an increase in labour market tightness also contributes to strengthen indirectly the informal sector. As a result, the final effect on the unemployment rate is *a priori* ambiguous:

$$\frac{\partial u}{\partial \theta} = -\frac{\delta \cdot g'(\theta)}{[\delta + g(\theta)]^2} - \frac{F'(R)\cdot[\delta + \rho + g(\theta) + 1] - F(R)\cdot g'(\theta)}{[\delta + \rho + g(\theta) + 1]^2}$$

with $R'(\theta) > 0$, and $F'(R) > 0$. It follows that the standard relationship between unemployment and labour market tightness may change. Indeed, if the informal sector is sufficiently large, then the *Beveridge Curve* of the economy switches from negative to positive:[11]

$$\frac{\partial u}{\partial \theta} > 0, \qquad \text{if } F(R) > \frac{\delta \cdot [\delta + \rho + g(\theta) + 1]^2}{[\delta + g(\theta)]^2} + \frac{F'(R)\cdot[\delta + \rho + g(\theta) + 1]}{g'(\theta)}$$

[11] According to Uren's (2007) calibrations: δ = 0.10, $q(\theta)$ = 4.5, $g(\theta)$ = 1.35, and θ = 0.3 which is consistent with the values of labour market tightness of this model. Furthermore, ρ is a tiny value. For instance, in Boeri and Garibaldi (2006) ρ = 0.06; whereas in Busato and Chiarini (2004) ρ = 0.03. Furthermore, $\lim_{\theta \to 0} g'(\theta) = \infty$. As a result, the *r.h.s.* of the inequality is a feasible value since it is lower than 1.

In a nutshell, an increase in labour market tightness increases the unemployment rate when the informal firms account for the better part of all economic activity. The rationale is the following: if the informal sector is sufficiently large, an increase in the probability to finding a job in the official sector moves workers from the sector which offers more job opportunities (informal) to the sector where there are less job opportunities (official).[12]

Finally, the empirical studies on the 'vacancies-unemployment' relationship don't take into account the underground economy, i.e. the informal vacancies. Unfortunately, the data to confirm a possible positive correlation between u and v where the informal sector is larger are scarce (the underground economy is a hidden phenomenon, by definition).

5. Conclusions

This paper develops a matching model of equilibrium unemployment, with on-the-job search, extended to the informal sector and to heterogeneous entrepreneurial ability. In this model, the entrepreneurial ability affects job productivity, all unemployed begin by searching in the official sector, and workers employed in the informal sector try to move into the official one. The model assumes the presence of a "search path" in which, at first, all unemployed workers search in the official sector and afterwards, in the case of failure, they search in the informal one. Basically, two key results emerge from this analysis:

- Firms become heterogeneous in productivity, thus providing a new solution to the problem of finding an interior equilibrium, where vacant jobs are allocated to both the regular and the hidden sector (the so-called *shadow puzzle*);
- If the informal sector is sufficiently large, an increase in labour market tightness increases the unemployment rate and then the 'vacancies-unemployment' relationship (the so-called *Beveridge Curve*) switches from negative to positive. In general, the ambiguity of this result reflects both the different trend of the two economic sectors and the intricate relationship found in the literature between unemployment and underground employment.

[12] Note that it is not a standard result of the assumption of on-the-job search. Indeed, in Boeri and Garibaldi (2002) the value of equilibrium unemployment monotonically falls with labour market tightness.

Appendixes

Appendix A: Properties of equation (2)

From the Bellman equations on the demand side it is straightforward to obtain:

$$V_r = -\frac{c}{\left[r+q(\theta)\right]} + \frac{q(\theta)\cdot(1-\beta)\cdot(xp-\tau)}{(r+\delta)\cdot\left[r+q(\theta)\right]}; \quad V_s = \frac{xp\phi - b - \rho\cdot\varphi\tau}{(1+r)\cdot\left[r+\delta+\rho+g(\theta)\right]}$$

hence, $V_r = -\frac{c}{\left[r+q(\theta)\right]} + \Omega(\theta)\cdot(xp-\tau); \quad V_s = \frac{xp\phi}{\Lambda(\theta)} - \frac{b+\rho\cdot\varphi\tau}{\Lambda(\theta)}$

since $\frac{q(\theta)\cdot(1-\beta)}{(r+\delta)\cdot\left[r+q(\theta)\right]} \equiv \Omega(\theta)$, and $(1+r)\cdot\left[r+\delta+\rho+g(\theta)\right] \equiv \Lambda(\theta)$. The threshold value R is a special x, so that it must be positive since $x \geq x_{min} > 0$. Sufficient conditions for the positivity of R are:

$$\frac{c}{\left[r+q(\theta)\right]} + \Omega(\theta)\cdot\tau > \frac{b+\rho\cdot\varphi\tau}{\Lambda(\theta)} \tag{A.1}$$

$$\Omega(\theta) > \frac{\phi}{\Lambda(\theta)} \tag{A.2}$$

In order to determine the restrictions on the parameters, we calculate the limit of conditions (A.1) and (A.2) for the labour market tightness which goes to zero: [13]

$$\tau\cdot\frac{(1-\beta)}{(r+\delta)} > \frac{b+\rho\cdot\varphi\tau}{(1+r)\cdot(r+\delta+\rho)} \tag{A.1'}$$

$$\frac{(1-\beta)\cdot(1+r)\cdot(r+\delta+\rho)}{(r+\delta)} > \phi \tag{A.2'}$$

since $lim_{\theta\to 0}\Omega = (1-\beta)/(r+\delta)$ by the *l'Hôpital rule*, and $lim_{\theta\to 0}\Lambda = (1+r)\cdot(r+\delta+\rho)$. Finally, we can use the mathematical limits to show that equation (2) is in fact increasing in θ: [14]

(i) $lim_{\theta\to 0}R \equiv \alpha > 0$, by conditions (A.1') and (A.2');

(ii) $lim_{\theta\to\infty}R = \frac{c/r}{0} \to +\infty$,since $lim_{\theta\to\infty}\Omega = 0$, and $lim_{\theta\to\infty}\Lambda = \infty$.

Note that $\alpha < x_{\max}$, since equation [2] has been built for $R \in [x_{min}, x_{max}]$.

Appendix B: Properties of equation (3)

The evolution of official employment in terms of the firm's transition rates is:

$$\dot{n}_r = \left[1 - F(R) - n_r\right]\cdot q(\theta) - \delta\cdot n_r$$

it follows that in steady-state, $\dot{n}_r = 0$, we get:

---

[13] The labour market tightness cannot be equal to ∞, since the total number of entrepreneurs in the economy is equal to 1 and because $u > 0$. Furthermore, the assumption that $\theta = \infty$ is often ruled out (see e.g. Fonseca et al., 2001; Pissarides, 2002) because in that case the value of a vacancy is negative.

[14] For the existence of an interior solution, the monotone nature of $R(\theta)$ is not necessary.

$$n_r = \frac{[1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}$$

Hence, the labour market tightness – i.e. equation (3) – can be rewritten as:

$$\left\{1 - \frac{[1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}\right\} \cdot \theta = [1 - F(R)] - \frac{[1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}$$

$$\left\{\frac{\delta + q(\theta) \cdot F(R)}{q(\theta) + \delta}\right\} \cdot \theta = \frac{[1 - F(R)] \cdot \delta}{q(\theta) + \delta}$$

$$\Rightarrow \theta \cdot \delta + g(\theta) \cdot F(R) = [1 - F(R)] \cdot \delta$$

since $\theta \cdot q(\theta) = g(\theta)$.

Hence, it is straightforward to obtain that for $F(R) = 1$ (all entrepreneurs enter the informal sector), i.e. $R = x_{max}$, then $\theta = 0$; whereas, for $F(R) = 0$ (none of the entrepreneurs enter the informal sector), i.e. $R = x_{min}$, then $\theta = 1$. Finally, totally differentiating the expression yields:

$$d\theta \cdot \delta + d\theta \cdot g'(\theta) \cdot F(R) + g(\theta) \cdot F'(R) \cdot dR = -F'(R) \cdot dR \cdot \delta$$

$$\frac{d\theta}{dR} = -\frac{F'(R) \cdot [\delta + g(\theta)]}{\delta + g'(\theta) \cdot F(R)} < 0, \text{ since } F'(R) > 0.$$

References

Albrecht, James, Lucas Navarro, and Susan Vroman, (2009), "The Effects of Labour Market Policies in an Economy with an Informal Sector," *Economic Journal*, 119(539), July, 1105-1129.

Audretsch, David B., and Max C. Keilbach, (2004), "Entrepreneurship Capital and Economic Performance," *Regional Studies*, 38 (8), 949-959, January.

Audretsch, David B., (2007), "Entrepreneurship Capital and Economic Growth," *Oxford Review of Economic Policy*, 23(1), 63-78, Spring.

Belke, Ansgar, Rainer Fehn, and Neil Foster, (2003), "Does Venture Capital Investment Spur Employment Growth ?," *CESifo Working Paper Series*, 930, March.

Boeri, Tito, and Pietro Garibaldi, (2002), "Shadow Activity and Unemployment in a Depressed Labour Market," *CEPR Discussion Papers*, 3433, June.

Boeri, Tito, and Pietro Garibaldi, (2006), "Shadow Sorting," *Fondazione Collegio Carlo Alberto Working Paper Series*, 10, May.

Bosch, Mariano, and Julen Esteban-Pretel, (2009), "Cyclical Informality and Unemployment," *CIRJE Discussion Papers*, 613, February.

Bouev, Maxim, (2002), "Official Regulations and the Shadow Economy: A Labour Market Approach," *William Davidson Institute Working Paper*, 524, December.

Bouev, Maxim, (2005), "State Regulations, Job Search and Wage Bargaining: A Study in the Economics of the Informal Sector," *William Davidson Institute Working Paper*, 764, April.

Busato, Francesco, and Bruno Chiarini, (2004), "Market and Underground Activities in a Two Sector Dynamic Equilibrium Model," *Economic Theory*, 23(4), May, 831-861;

Dulleck, Uwe, Paul Frijters, and Rudolf Winter-Ebmer, (2006), "Reducing Start-up Costs for New Firms: The Double Dividend on the Labour Market," *Scandinavian Journal of Economics*, 108(2), 317-337, 07.

Fonseca, Raquel, Lopez-Garcia Paloma, and Christopher A. Pissarides, (2001), "Entrepreneurship, start-up costs and employment," *European Economic Review*, 45(4-6), May, 692-705.

Fugazza, Marco, and Jean-Francois Jacques, (2004), "Labour market institutions, taxation and the underground economy," *Journal of Public Economics*, 88(1-2), January, 395-418.

Kolm, Ann-Sofie, and Birthe Larsen, (2003), "Wages, Unemployment, and the Underground Economy," *CESifo Working Paper*, 1086, November.

Kolm, Ann-Sofie, and Birthe Larsen, (2010), "The Black Economy and Education," *Research Papers in Economics*, 2010 : 3, Stockholm University, Department of Economics.

La Porta, Rafael, and Andrei Shleifer, (2008), "The Unofficial Economy and Economic Development," *NBER Working Paper*, 14520, December.

Lisi, Gaetano and Maurizio Pugno (2010), "Entrepreneurship and the Hidden Economy: An Extended Matching Model", *International Economic Journal*, 24(4), 587-605.

Mortensen, Dale T. (2005), "Alfred Marshall Lecture: Growth, Unemployment, and Labor Market Policy", *Journal of the European Economic Association*, 3(2-3), 236-258, 04/05.

Petrongolo, Barbara, and Christopher A. Pissarides, (2001), "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), June, 390-431.

Pissarides, Christopher A., (2000), *Equilibrium Unemployment Theory* (2nd Edition), The MIT Press.

Pissarides, Christopher A., (2002), "Company Start-Up Costs and Employment," *CEP Discussion Papers*, dp0520, March.

Tanzi, Vito, (1999), "Uses and Abuses of Estimates of the Underground Economy," *The Economic Journal,* 109(46), June, 338-347.

Uren, Lawrence, (2007), "Entrepreneurship and Labour Market Fluctuations," *Economics Bulletin*, 10(13), 1–11.

Zenou, Yves, (2008), "Job Search and Mobility in Developing Countries. Theory and Policy Implications," *Journal of Development Economics*, 86(2), 336-355, June.

Figures

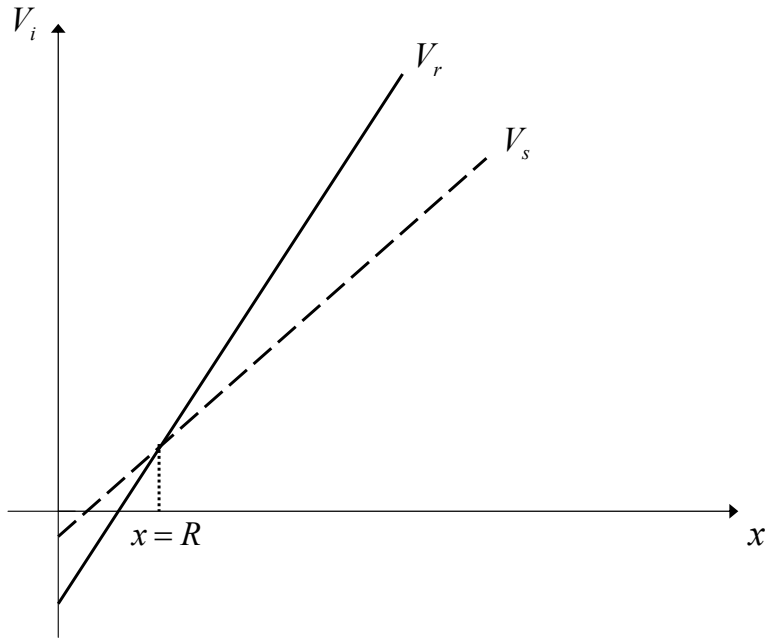

Figure 1. Entrepreneurs' indifference condition

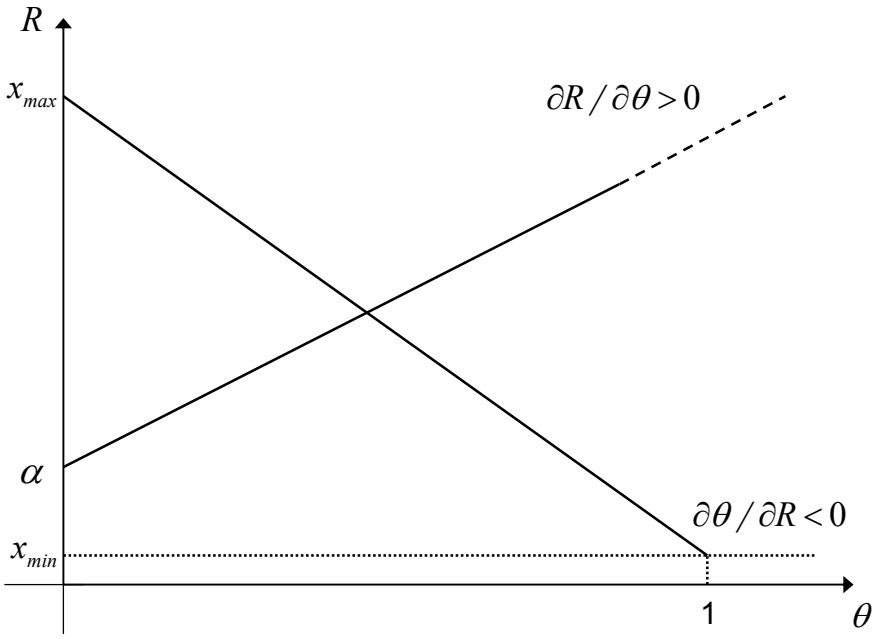

Figure 2. Equilibrium interior solution

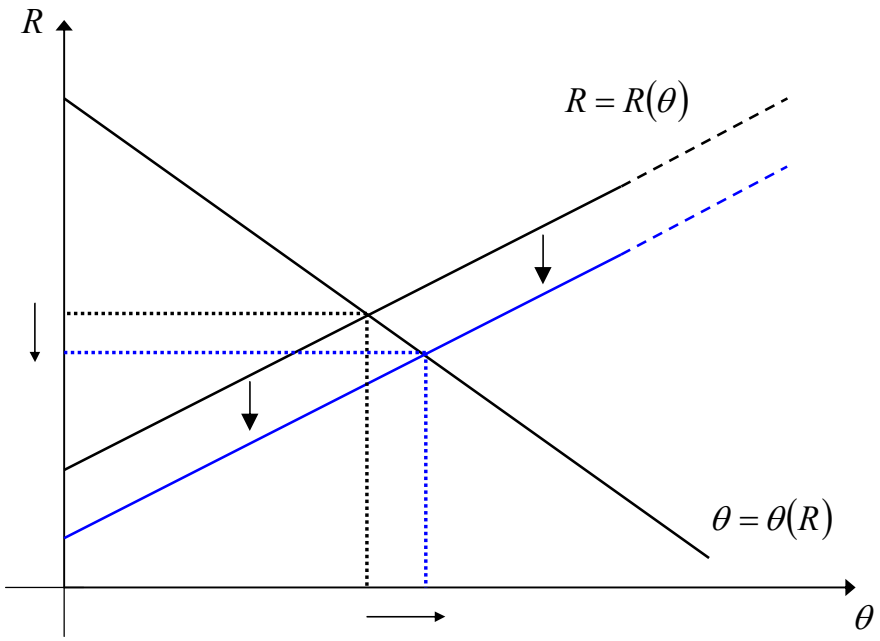

Figure 3. Increase in productivity